

Investigation of Un-signalized Roundabouts Delay with Adaptive-Network-Based Fuzzy Inference System and Fuzzy Logic

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Keywords	Abstract
Un-signalized Roundabout, Delay, Fuzzy Logic, Adaptive-Network-Based Fuzzy Inference System, Artificial Neural Network.	A lot of models have been suggested for calculation and estimation of vehicles delay of signalized and un-signalized intersections. The geometry of intersection and traffic flow can affect delay imposed on vehicles. The city of Rasht, according to its own special characteristics, has a large number of un-signalized roundabouts; therefore, investigation of delay of these roundabouts could be an important issue. The objective of this paper is to use two methods fuzzy logic and adaptive-network-based fuzzy inference system to obtain the values of optimal delay in un-signalized roundabout. Based on the analysis of prediction model results, fuzzy logic and adaptive-network-based fuzzy inference system methods, the results indicate that both methods can be used for modeling and predicting the delay of un-signalized roundabouts under varying traffic parameters.

1. Introduction

Intersections play a key role in the performance of the road network. Recently, there has been growing concern in the choice of roundabouts for control of intersections. The use of modern roundabouts significantly decreases severe injury and fatality crashes and allow drivers to get through intersections more quickly. In addition, roundabouts are safer way for pedestrians and bicyclists for navigating traffic [1, 2].

Nowadays, traffic delay is an evaluating criterion of controlled intersection efficiency by stop sign and yield sign, signalized intersections and roundabouts [3]. Delay is generally defined as the difference between the time assigned for passing a road section under ideal assumptions and the actual travel time [4]. Different models have been presented for estimating vehicle delays of signalized and un-signalized intersections. One of the first models of delay estimation was suggested by Wardrop assuming that vehicles arrive at the intersection at the same time. Kimber et al. suggested correlations between delay and traffic intensity [5]. Troutbeck derived an equation for un-signalized intersections as a function of several parameters such as Adam's delay which is defined as an average delay when the minor stream flow is low, the degree of saturation of the minor stream (minor stream entry flow/maximum entry flow) and a form factor (which quantifies the effect of queuing in the minor stream) [6].

As the demand for construction of roundabouts increases, there is a growing need to analyze roundabout capacity and delay. Analysis of roundabouts can be achieved by using empirical and analytical methods. Empirical method can be achieved by field data collection in order to assess the correlation between geometric parameters and performance factors like capacities and delays. The analytical models rely on theoretical understanding about driver behavior and vehicle performance at the intersection.

Flannery et al. [7] derived an analytical model for stopped time delay estimation at single lane roundabouts by using Little's law as

$$L_q = \lambda E(T) + \frac{\lambda^2 [E(T)]^2 + VAR(T)}{2(1 - \lambda E(T))} \quad (1)$$

where $E(T)$ is the expected service time, $VAR(T)$ is the variance of a random variable (T), L_q is queue length, and λ is the mean arrival rate to the queue. Al-Omari et al. [8] derived an empirical model as a function of impressing parameters, according to the time interval of 15 min as follows

$$D_s = 0.0027V_s + 0.0056V_c - 0.1802D_i + 0.8048W_c - 0.3083W_e \quad (2)$$

in which, D_s is stopped delay (s/veh), V_s is volume of vehicles in the subject entry (pcuph), V_c is volume of vehicles in the circulating roadway (pcuph), D_i is diameter of the

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roundabout island (m), W_c is width of the circulating roadway (m), and W_e is width of the subject approach entry (m). Roundabout delays can be divided in to two section for each entry approach. Queuing and geometric delay are two distinct components. Geometric delay outcomes from vehicles slowing down when passing the roundabout. Queuing delays happen when the drivers are waiting for proper gap. Harging showed that geometric delay is a function of approach speed assuming that the acceleration and deceleration rates are equal as following [9]:

$$d_g = 0.0012v^2 + 0.0254v + 1.5 \quad (3)$$

Here, d_g is the average vehicle geometric delay (sec), and v is average approach speed (km/hr). There are several studies have been done concerning the delay of signalized intersection in the city of Rasht [10, 11]. In spite of the fact that various un-signalized roundabouts exist in the Rasht, few researchers have studied about geometric and traffic parameters of roundabouts. According to this, Bargegol et al. suggested a delay model of un-signalized roundabouts by using neural network and regression [12]. The objective of this paper is to use two methods of fuzzy logic and adaptive-network-based fuzzy inference system (ANFIS) to obtain the values of optimal delay in un-signalized roundabout. Then comparison these two methods with each other and artificial neural network (ANN) to choose the best model to determine the optimal delay values are in un-signalized roundabout.

2. Delay Criterion

Traffic delay is one of the evaluating criteria of controlled intersections' function with stop signs, yield signs, signalized intersections and roundabouts. The majority of studies which have been done for development of delay model at controlled intersections are with stop signs and traffic lights, and roundabouts have not been paid much attention. In HCM 1994, delay model is for controlled intersections including stop signs and traffic lights, but no models are considered for roundabouts. In both HCM 1997 and HCM 2000, there is a method for estimate of roundabout capacity, although there are no models considered for the estimate of delay in roundabout [13 -15].

But delay relation on the vehicles which has been mentioned in HCM 2010 has many nearside variables for calculation of vehicles delay such as cycle length, green time, saturation flow rate, capacity, etc. In this paper, assuming all other variables constant, volumes of nearside vehicles to intersection is just assumed as a variable. Any random variable has a specified frequency distribution with given mean and variance that is one of the characteristics of every distribution frequency [16].

Assuming mean and variance of nearside volume and nearside distribution of vehicle to intersections, the values are clear, it is possible to calculate mean and variance of saturation degree by Eq. (4), on the basis of Eq. (4), the type of frequency distribution of vehicles' saturation rate would be equal with the type of frequency distribution of vehicles' nearside volume to intersection.

$$X = V/C \quad (4)$$

where X is the saturation flow rate of near side route to the intersection, V the nearside flow rate of the vehicles to the

intersection and C the capacity of near side route to the intersection. Given that saturation flow rate, green time of each phase and cycle length are constant, so with the above assumptions the capacity of each nearside route to intersection on the basis of Eq. (5) will be calculated as

$$C = S \times \frac{g_i}{c} \quad (5)$$

In the above relation, C stands for the capacity of each nearside route to the intersection, S is the saturation flow rate, g_i is the green time related to the intended phase and c denotes the cycle length of the traffic light. According to Eq. (1), the average level of saturation rate which is equal to the level of nearside flow rate of the vehicles to the intersection, will be divided by the capacity of nearside route to intersection. Also, on the basis of Eq. (2), the variance of saturation rate which is equal to the variance of nearside flow of the vehicles to intersection is divided by the square of the capacity.

Eq. (6) is HCM 2010 equation of delay function of the vehicles. After the calculation of the expected values of saturation rate, the amount of vehicles' delay on the basis of HCM delay function with the help of Taylor series will be estimated. On the basis of assumptions made in this paper based on Eq. (6), the saturation flow rate is the only random variable [17]:

$$D = 0.5C \times \frac{(1-\frac{g}{c})^2}{(1-\text{Min}(x,1)\frac{g}{c})} + 900T \left[(x-1) + \sqrt{(x-1)^2 + \frac{8kix}{cT}} \right] + d_3 \quad (6)$$

where C, g, x and c are the cycle length, green time related to the intended phase, saturation degree and capacity, respectively. Delay at the intersections in comparison with roundabout capacity is not paid much attention by the researchers.

3. Case Study

In this study three un-signalized roundabouts with different features were chosen among 10 four leg roundabouts in Rasht. Weather condition and observation time were the same for all of the roundabouts. It was tried to select various roundabouts in terms of the street ending to them and their location in the city of Rasht. The properties of the roundabouts are presented in Table 1.

In order to achieve objective of this study, data of traffic volume and geometric features and data of green time at some of un-signalized roundabouts have been collected. The video recording technique was used for volume data collection during noon peak hour periods. All of the travel times of vehicles during the whole observation time (measured by a chronometer) were collected by the video films.

4. Discussion

4.1. Artificial Neural Network

In this research study, ANN model was trained using back propagation (BP) algorithm. There are various ways to minimize the error function; in this study, Levenberg Marquardt algorithm was used for this purpose. In the ANN model, created in MATLAB, 60% of data was used for training, 20% for cross-validation and 20% as test data. In order to determine the range of normalization of data, the

data set was mapped into the range of (0, 1) and log-sigmoid function was used as the activation function. Number of neurons in the hidden layer are significantly important, especially for multilayer perceptron networks (MLP) that are used in solving engineering problems. This phase was repeated 10 times for each number of neurons and the best result was recorded. The previous phases were then repeated

with 2 hidden layers and after comparing the statistical results of the two phases, the optimum network structure was selected [21].

Table 1. Cross-section characteristic of study place

Roundabout	Time of Observation (hour)	Enter Name	Cross-section characteristic						
			Nearside			Roundabout Diameter (meter)	Far-side		
			Parking Lane	Passing Lane	Total		Parking Lane	Passing Lane	Total
1	2	Shariati St.	2.2	5.8	8.00	16	2.2	7.80	10.00
		Motahari St.	2.2	7.8	10.00		2.2	6.80	9.00
		Ehsanbakhsh St.	2.2	11.3	13.50		2.2	8.80	11.00
		Shoha St.	2.2	11.1	13.30		2.2	14.80	17.00
2	1	Golsar St.	2.2	9.05	11.25	28	2.2	9.05	11.25
		Saady St.	2.2	6.55	8.75		2.2	6.55	8.75
		Ansari St.	2.2	8.80	11.00		2.2	8.80	11.00
		Takhti St.	2.2	9.55	11.75		2.2	9.55	11.75
3	2	Emam St.	2.2	16.40	23.00	90	2.2	9.30	9.30
		Emam Ali St.	2.2	8.40	10.60		2.2	8.40	10.60
		Khalje Fars St.	2.2	6.55	8.75		2.2	6.55	8.75

Evaluating means becoming confident about the ability of the model to generalize the results obtained from the train set. Squared correlation coefficient (R^2), root mean square error (RMSE) and coefficient of variation (COV) were used for this purpose in this study. In Eqs. (7)-(9), Y^{mea} , Y^{pre} and \bar{Y}^{mea} are observed values, estimated values and the average of observed values, respectively ([22]).

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (Y^{pre} - Y^{mea})^2}{n}} \quad (7)$$

$$R^2 = 1 - \frac{\sum_{i=1}^n (Y^{mea} - Y^{pre})^2}{\sum_{i=1}^n (Y^{mea})^2} \quad (8)$$

$$COV = \frac{RMSE}{|\bar{Y}^{mea}|} \times 100 \quad (9)$$

In order to estimate delay of un-signalized roundabouts, ten neural networks with different architectures were designed and tested based on the RMSE, COV and R^2 values (Table 2). The architecture of ANN selected in this study for un-signalized Roundabouts delay simulation is the multilayer feed forward net trained by back propagation with one layers of hidden units 25 neurons. The output units and the hidden units have biases. The output layer has one neuron. All the analyses were performed with the use of Matlab software. Initially the ANN was trained only by the actual data and then, the obtained square mean deviation between the actual data and results obtained by the ANN was very small, presenting that the method describes the actual data with great accuracy. As seen in Table 2, training accuracy improves by increasing the number of neuron until 25 as indicated by the smaller RMS and COV values and R^2 values approaching 1. Figures 1 and 2 show the neural network results related to the training performance and training regression, respectively. In these Figures, the parameter Mu represents the momentum update of the network and the parameter epoch shows the measure of the number of times all of the training vectors are used once to

update the weights. Based on the result, ANN training performance reveals the amount of gradient, MU (momentum update) and validation fail at epoch 5 resulted 5.7198×10^{-12} , 1×10^{-8} and 2, respectively. In addition, the best validation performance based on the mean square error (MSE) value is 0.012849 at epoch 3.

4.2. Application of Fuzzy Logic

In this study, a fuzzy logic approach was adopted for modeling in conditions of limited information content. The fuzzy logic transforms the verbal, qualitative expressions in numeric values and has wide application in modeling of various systems [23]. The process of fuzzy inference contains of the pieces that are membership functions, fuzzy logic operators and if-then rules. There are two types of fuzzy inference systems that are Mamdani type and Sugeno-type. Mamdani fuzzy inference system was used in this research for modeling the delay of un-signalized roundabouts [20]. Fuzzy approach considers cases where linguistic uncertainties play some role in the control mechanism of the phenomena concerned. Fuzzy propositions, i.e. IF THEN statements are used to properties the state of a system and the truth value of the proposition is a measure of how well the description matches the state of the system [18,19,24]. Figure 3 shows the fuzzy inference system for the delay of un-signalized roundabouts. The influence of the varying traffic parametric such as right turning volume (RTV), left- direct and turning volume (LDTV), roundabout turning volume (RBTV), fare side volume (FV) and roundabout diameter (RD) on the delay of the un-signalized roundabouts was tried to modelling by using the fuzz logic method. Using the fuzzy logic, the RTV, LDTV, RBTV, FV and RD with reasonable precisions are all systematically varied to identify the effects of each combination for finding the delay values of un-signalized roundabouts.

Membership functions are the fundamental aspects of fuzzy systems. The membership functions assumed for the entrance variables are shown in Figure 4. For example, for Right Turning Volume (RTV) parameter, seven fuzzy sets were assumed; for MIN, VLRVT, LRVVT, MRVT, HRVT, VHRVT and MAX, were used. The symmetric Gaussian member-ship functions were used.

The degree of memberships “1” was shown for input and output parameters. On the membership function graphics, “y”-axis value is maximum (1= 1) and minimum (1= 0) for temperature, exposure time and stability. There are MIN, VL, L, M, H, VH and MAX on the membership

Table 2. Statistical Results of the Delay Prediction Model for Different Network Architectures

Network structure	RMSE	COV	R ²	R Value			
				Training	Validation	Testing	All
8-2-1	40.01	73.44	0.7210	0.7553	0.7822	0.4754	0.7227
8-4-1	11.93	22.30	0.9590	0.9486	0.8735	0.5871	0.8799
8-5-1	13.79	25.78	0.9452	0.8976	0.5690	0.6145	0.8339
8-7-1	10.67	19.94	0.9672	0.9808	0.8960	0.2361	0.9046
8-8-1	11.75	21.95	0.9603	0.9623	0.9510	0.0965	0.8906
8-10-1	12.84	24.00	0.9525	0.8435	0.9311	0.0920	0.8643
8-15-1	12.64	23.62	0.9540	0.9969	0.5188	0.6209	0.8748
8-20-1	13.42	25.09	0.9481	0.9911	0.8511	0.6165	0.8894
8-25-1	10.18	19.028	0.9701	0.9999	0.8700	0.9220	0.9265
8-30-1	11.42	21.09	0.9643	0.9653	0.9520	0.0945	0.9004

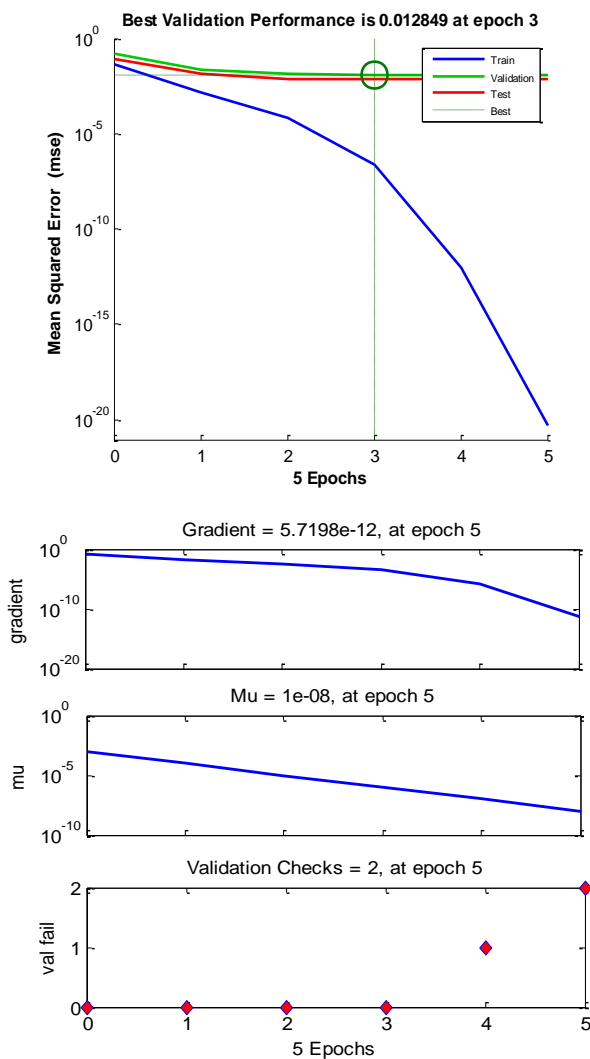


Figure 1. Neural network training performance.

logic. These if-then rule statements are used to formulate the conditional statements that comprise fuzzy logic. After the definition of membership functions, a system of ‘rules’ was chosen. In addition, fuzzification process applies the fuzzy logic operators and resolves the premise to a single number between 0 and Mamdani type of a fuzzy inference system was used. A single output fuzzy set, obtained by aggregating the output fuzzy sets for each rule, was defuzzified, receiving a single number.

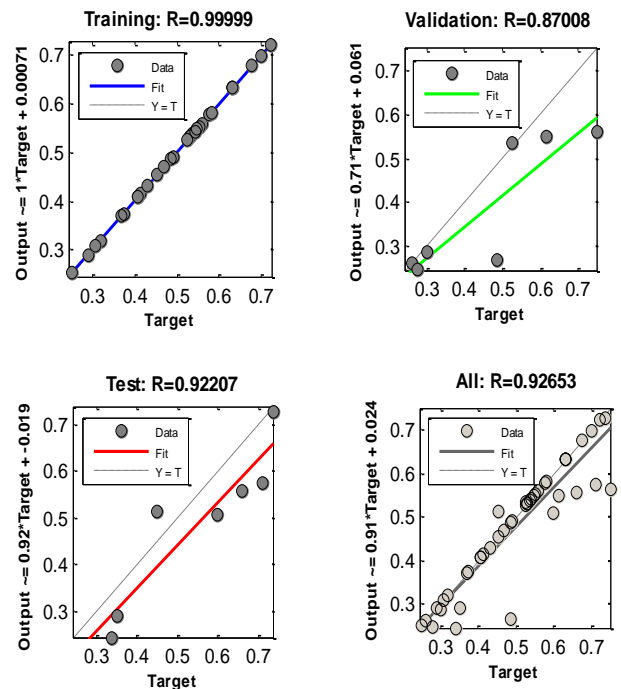
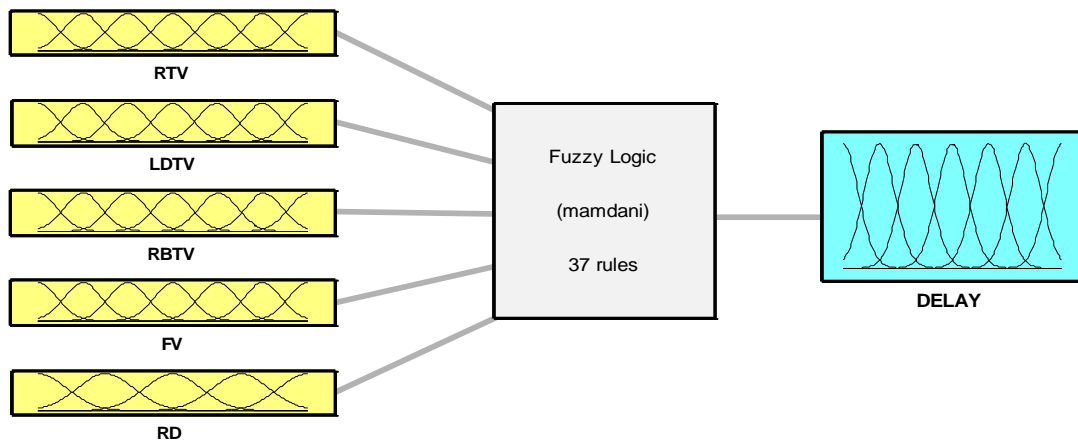


Figure 2. Neural Network Training Regression.

functions, fare side volume and roundabout diameter. Fuzzy sets and fuzzy operators are the subjects and verbs of fuzzy



System Fuzzy Logic: 5 inputs, 1 outputs, 37 rules

Figure 3. Fuzzy inference system for the delay of un-signalized roundabouts

4.3. Adaptive-Network-Based Fuzzy Inference System

ANN and fuzzy logic are used in ANFIS’s architecture [17]. ANFIS, which is used learning algorithms of neural network, is consisted of if-then rules and couples of input–output. The aim of system modeling is that it can be used in computer simulations compared to physical systems which are used in real applications. In this way, practical applications can be realized simply. In this study, the right turning volume, left-direct and turning volume, roundabout turning volume, fare side volume and roundabout diameter are the parameters chosen as the input layer and the un-signalized roundabouts delay as the output layer. Figure 5 shows the ANFIS system for the delay of un-signalized roundabouts. In this research study, hybrid learning algorithm for ANFIS and two, three and four input membership functions for each of all inputs has been used. These input membership functions are triangular, trapezoidal, gbell, gauss 2, and gauss.

The dataset for the un-signalized roundabouts delay of system available included 44 data patterns. In the prediction model, 80% of data was used for training the model and 20% of data, which were completely separate from train data, were used to test and validate the model. ANFIS topologies with various input membership functions and number of input membership functions are trained.

As seen in Table 3, training accuracy improves by decreasing the number of input membership functions as indicated by the smaller RMS and COV values and R² values approaching 1. On the other hand, beyond a certain point the errors obtained begin to increase together with the complexity of the ANFIS as the larger the number of input membership functions the more complex the network is. Based on the statistical data presented in Table 2, for delay values of algorithm by using two gauss input membership functions appeared to be most optimal topology. This topology gained 4.1144 mean RMS value, 7.6872 mean COV value and, 0.9952 mean R² value. Figure 6 shows the membership functions of ANFIS approach for the delay of un-signalized roundabouts. In addition, the surface viewer was used to display the dependency of the delay on the right turning volume, left-direct and turning volume, and roundabout turning volume, fare side volume and

roundabout diameter. The surface viewer indicates the behavior of the entire system in Figure 7.

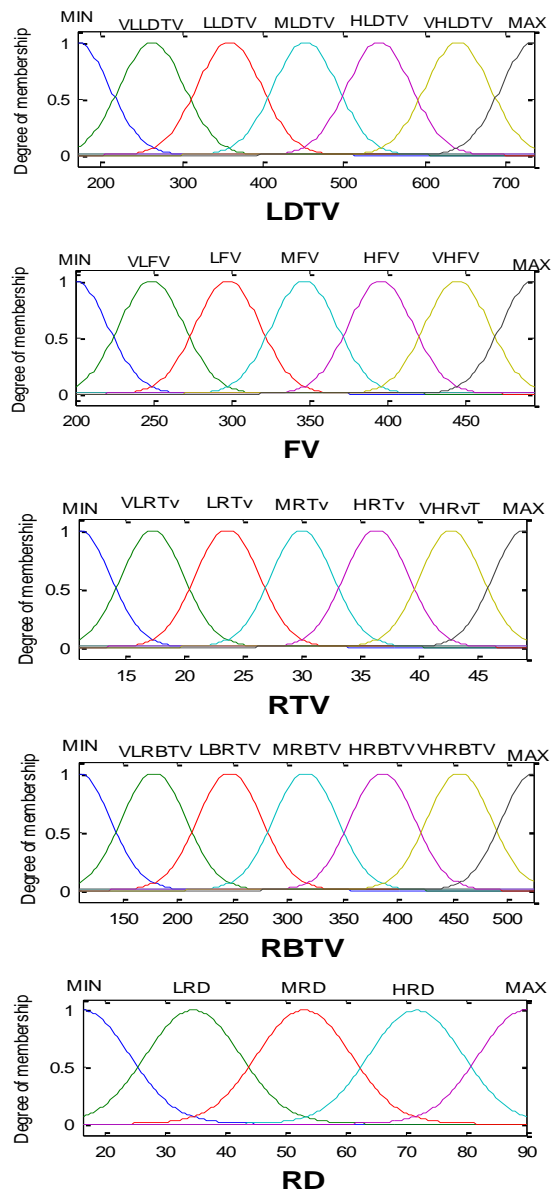


Figure 4. Membership functions of Fuzzy Logic system for the delay of Un-signalized Roundabouts.

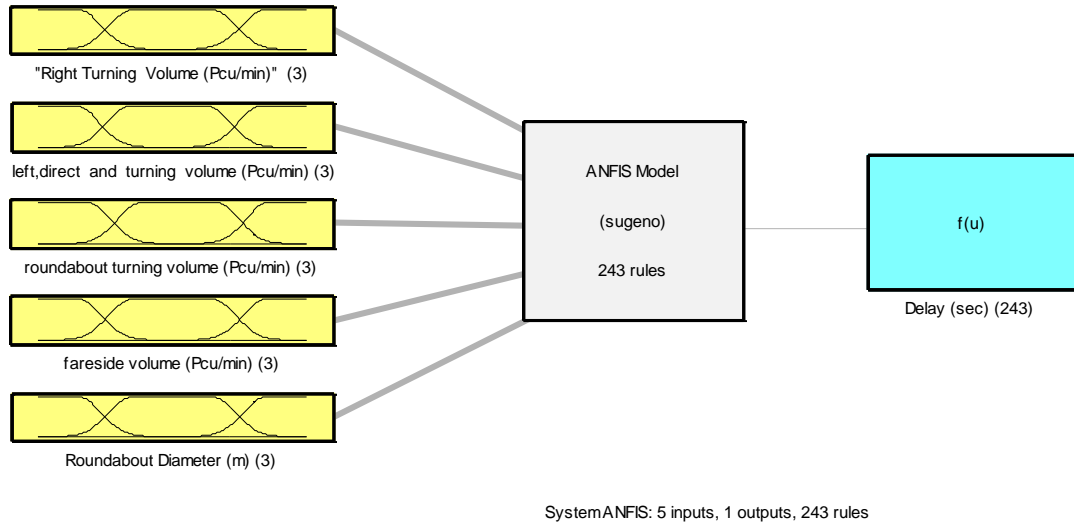


Figure 5. ANFIS system for the Delay of Un-signalized Roundabouts.

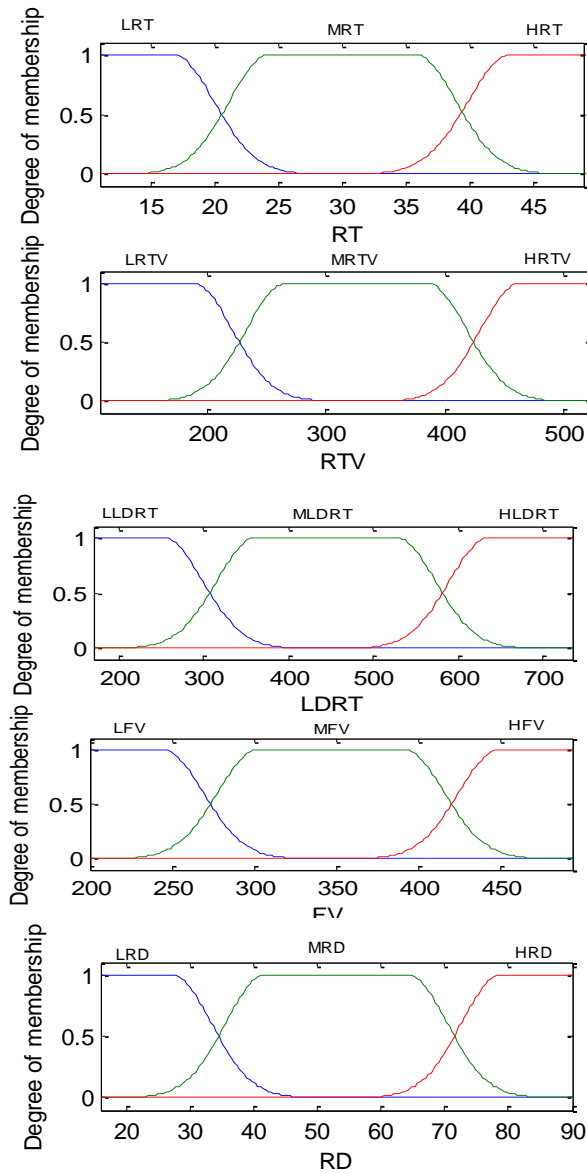


Figure 6. Membership functions of ANFIS approach for the delay of Un-signalized Roundabouts.

Table 3. Statistical values of ANFIS model

Algorithm- input membership functions	Training			Testing			All		
	RMS	COV	R ²	RMS	COV	R ²	RMS	COV	R ²
Two triangular	4.4385	8.4453	0.9941	30.2338	52.2398	0.8078	12.8568	24.0212	0.9536
Three triangular	0.0047	0.0090	0.9994	52.1963	90.1879	0.3451	22.2566	41.5834	0.8576
Four triangular	0.0003	0.0006	0.9999	40.3099	69.6500	0.6583	16.2640	30.3870	0.9257
Two Trapezoidal	7.3932	14.0674	0.9836	37.7306	65.9050	0.6477	17.6243	33.5776	0.9069
Three Trapezoidal	0.0030	0.0058	0.9994	26.2294	45.8156	0.8297	11.3136	21.5544	0.9616
Four Trapezoidal	2.0138	3.8318	0.9988	40.8134	71.2898	0.5878	17.6582	33.6421	0.9065
Two gbell	4.7119	8.9655	0.9933	36.6015	63.9328	0.6685	16.3652	30.5762	0.9248
Three gbell	0.0054	0.0102	0.9991	39.0442	68.1994	0.6228	16.8410	31.4651	0.9203
Four gbell	0.0004	0.0008	0.9998	34.4774	60.2225	0.7058	14.8712	28.3323	0.9337
Two gauss	4.4835	8.5309	0.9940	73.7253	128.7778	0.3451	32.0635	59.9064	0.7112
Three gauss	0.0069	0.0132	1.0000	21.6929	37.8916	0.8835	9.3568	17.4820	0.9754
Four gauss	0.0005	0.0010	1.0000	54.0033	94.3290	0.2783	23.2933	44.3781	0.8374
Two gauss2	5.1525	9.8040	0.9920	48.1860	84.1676	0.4254	21.3121	39.8188	0.8724
Three gauss2	0.0062	0.0117	0.9997	9.5389	16.6618	0.9775	4.1144	7.6872	0.9952
Four gauss2	0.0012	0.0024	0.9998	30.3385	52.9930	0.7722	13.0859	24.9311	0.9487

4.4. Comparison of ANN, Fuzzy Logic and ANFIS Systems

Figure 8 shows a comparison of between measured delay and prediction values with ANN, Fuzzy Logic and ANFIS systems. On these figures 1:1 line has also been indicated for visual comparison. Comparing the measured delay values with those obtained from ANN, ANFIS and Fuzzy models it is obvious that they are similar. Besides the ANN, ANFIS and fuzzy model results are very close values to each other; there is little difference. These all assert that the measured delay and the ANN, ANFIS and fuzzy models results are in harmony. Based on the results, the ANFIS, ANN and fuzzy models showed the highest performance in predicting delay, respectively. Furthermore, In Figure 9, a point to point comparison is made between measured and predicted delay values by the ANFIS, ANN and Fuzzy models. Based on the results, there is less difference between predicted and measured delay in the ANFIS, ANN and Fuzzy models.

Table 4 shows the results for error rates and statistical values of the ANN, Fuzzy Logic and ANFIS results on delay of un-signalized roundabouts. According the results,

the absolute error maximum of Fuzzy value is 26.48, absolute error average is 12.41 and its average percent error is 26.27%. In addition, ANN model error rates reveal the amount of maximum absolute error, absolute error average and average percent error resulted 39.18, 4.47 and 9.12%, respectively. Finally, investigation of ANFIS model on delay prediction shows that, amount of maximum absolute error, absolute error average and average percent error resulted 17.30, 1.7 and 4.87%, respectively. Based on the results, when the error parameters of the results from ANN, fuzzy logic and ANFIS results are compared it is clear that the error averages of the ANFIS model results are lower. In the other hands, the ANFIS model results are more similar to the measured delay results. Furthermore, to study the prediction performance of each delay model at un-signalized roundabouts the RMS, COV and R² values were calculated. Based on the results, statistical values accuracy were improved by using the ANFIS model as indicated by the smaller RMS and COV values and R² values approaching 1. In fact, the best prediction performance based on RMS, COV and R² values are 4.11, 7.69 and 0.9952, respectively.

Table 4. The error rates and statistical values of the ANN, Fuzzy Logic and ANFIS results

Statistical values	Fuzzy	ANN	ANFIS
Max absolute error	26.48	39.18	17.30
AVE absolute error	12.41	4.47	1.70
AVE percent error (%)	26.27	9.12	4.87
RMS	14.17	10.18	4.11
COV	26.48	19.03	7.69
R ²	0.9423	0.9702	0.9952

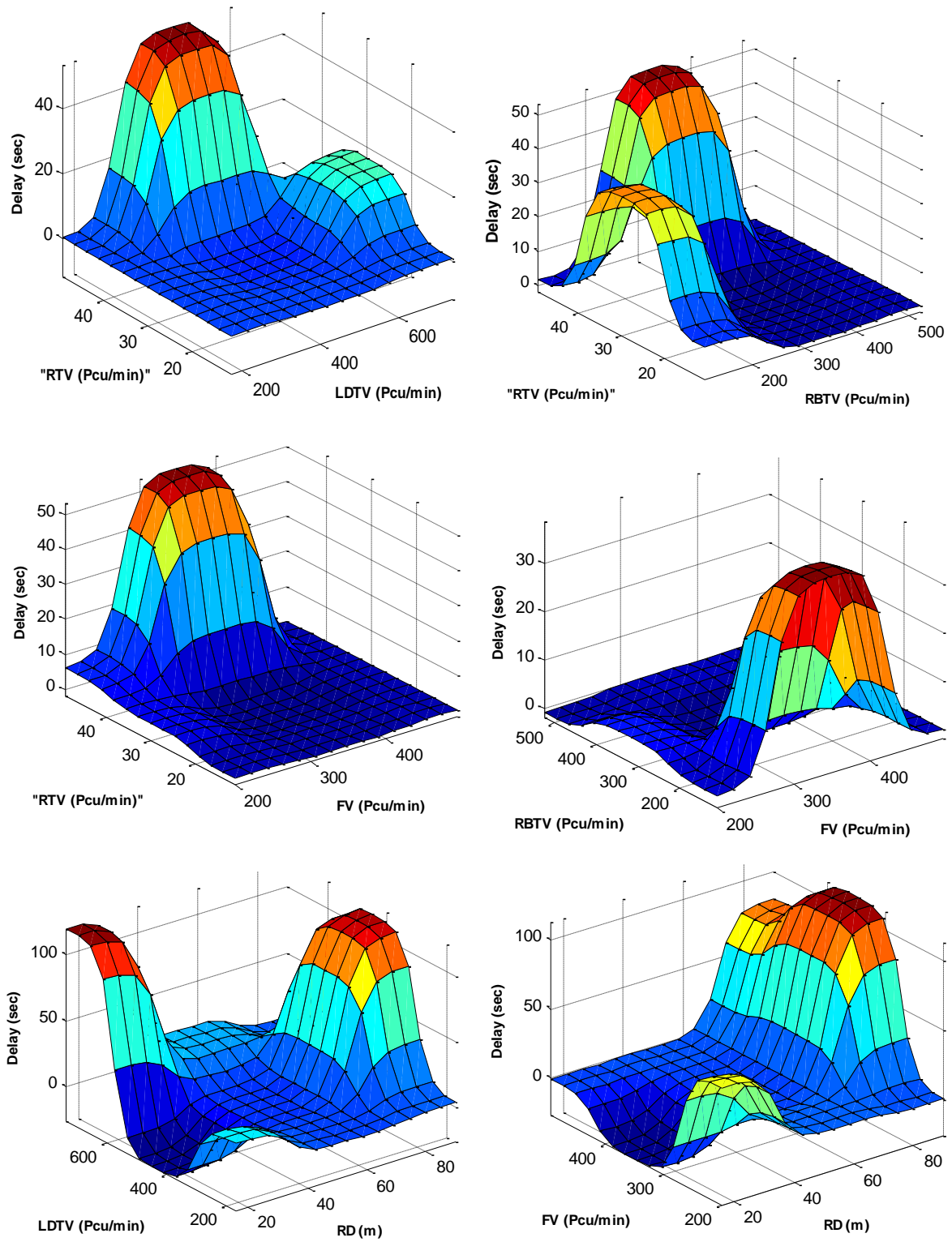


Figure 7. The surfaces viewer of ANFIS system for the delay of un-signalized roundabouts

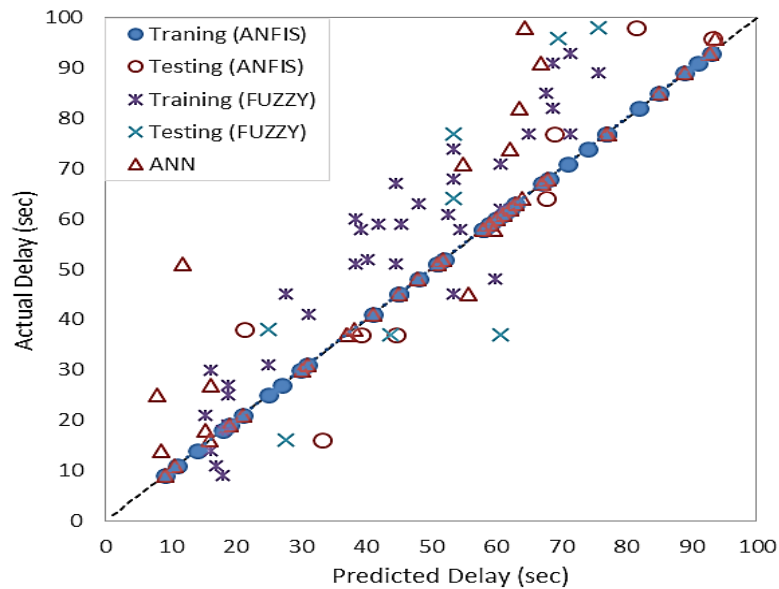


Figure 8. Relationship between measured delay and prediction values with ANN, Fuzzy Logic and ANFIS systems

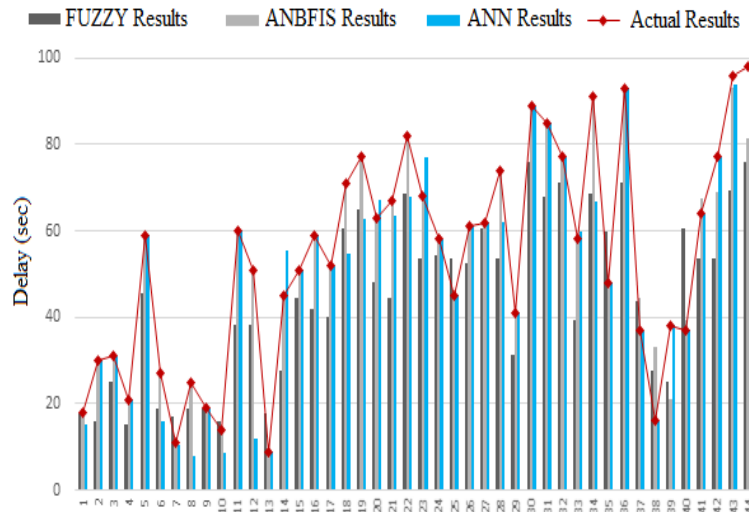


Figure 9. The comparison of actual, ANN, fuzzy logic and ANFIS predicted delay of un-signalized roundabouts

8. Conclusions

This study focused on the investigation of un-signalized roundabouts delay with ANFIS, ANN and FL. Based on the analysis of prediction model results, following findings were concluded:

- The result of ANN model shows that the training accuracy was improved by increasing the number of neuron until 25 as indicated by the smaller RMS and COV values and R^2 values approaching 1.
- Fuzzy logic and ANFIS methods analysis indicates that the both methods can be used for modeling and predicting of the delay of un-signalized roundabouts under varying traffic parametric.
- A comparison of delay predicted by the ANN, ANFIS and fuzzy logic models and delay measured showed a high R^2 and low RMSE in all three models. However, it is demonstrated that

ANFIS is an excellent method that can be used as a tool in investigating the factors affecting delays of un-signalized roundabouts.

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